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Multiphoton 2D exciton absorption in a superlattice

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Abstract. Multiphoton exciton transitions induced by an intense optical wave in a semiconductor superlattice are analysed. It is assumed that the excitons can be treated as two-dimensional (2D) excitons localized in the heteroplanes and that the oscillating electric field of the optical wave is directed perpendicular to the heteroplanes. The superlattice potential barriers are modelled by a periodic chain of δ-type functions. Quasienergetic time-dependent states are used. The explicit dependence of the coefficient of the multiphoton absorption on the frequency and magnitude of the optical wave and on the superlattice and exciton parameters is obtained. The exciton absorption spectrum is shown to consist of a series of continuous absorption bands associated with the discrete levels of the exciton. The width of each band is equal to the sum of the widths of the electron and hole minibands. The distance between the neighbouring bands is equal to the separation of the exciton levels. The form of the exciton absorption essentially depends upon the parity of the number of absorbed photons and on the relationship between the separation of the exciton levels and the total width of the electron and hole minibands.

1. Introduction

At the present time, there are many publications devoted to the optical properties of semiconductor superlattices. The most common example is that of the heterostructure formed by alternating layers of GaAs and GaAlAs semiconductors, which have different forbidden gaps. It is possible to select two kinds of potential function that define the electronic properties of superlattices. The first is a superlattice potential, which influences the electron and hole motion in a direction Oz normal to the heterolayers. This potential consists of a periodic sequence of potential wells separated by barriers. The energy spectrum associated with this potential consists of a series of allowed and forbidden minibands. (The detailed description of the miniband states is given in standard texts [1, 2].) The second potential is that of the Coulomb interaction between an electron in the conduction band and a hole in the valence band. This interaction is responsible for the bound state of an electron–hole pair, also known as a Mott exciton. If the effective Bohr radius of the exciton is greater than the size of the superlattice well, the exciton has a two-dimensional (2D) character and its states are localized in the heteroplanes. For example, these excitons can be realized in a GaAs/GaAlAs superlattice if its period is less than about 100 Å. At the present time, excitonic effects are observed in the majority of III–V and II–VI semiconductors.

Both established [3, 4] and more recent [5, 6] experimental measurements show that excitons play an important rôle in the formation of the absorption and luminescence spectra

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of quasi-two-dimensional systems. It is necessary therefore to take excitonic effects into account in any theory of optical phenomena in real semiconductor heterostructures. It has been shown theoretically [7] that the oscillator strengths of optical transitions within states of the 2D exciton are large. Exciton states in a superlattice have been calculated previously [8] both without and with external fields [9]. A detailed review of theoretical work devoted to exciton effects in quasi-two-dimensional structures has appeared recently [10]. Usually, either variational- or standard numerical-type calculations of the one-photon exciton absorption in a superlattice of a weak optical wave have been carried out. It appears that a general analytical consideration of multiphoton exciton absorption in a superlattice induced by an intense optical wave has received little attention. However, an analytical approach to the calculation of the interband one-photon [11, 12] and multiphoton [13] optical transitions in a semiconductor superlattice has been developed very recently by some of the present authors.

The aim of this paper is to extend the technique used previously [13] to the exciton problem. An analytical theory of multiphoton exciton absorption in a semiconductor superlattice is developed below. The oscillating electric field of the optical wave is along the Oz direction of the superlattice. The transverse states of the carriers in the x–y plane are defined by the Coulomb electron–hole interaction; they are the 2D exciton states. The theory is based on the assumption that the real superlattice potential is replaced by the limiting form of the Kronig–Penney potential in which the barriers are represented by a periodic chain of δ-type functions. The method uses quasienergetic time-dependent states for the motion along Oz. These states take into account the effect of the large oscillating electric field on the carriers.

2. Background

We consider first interband optical absorption (OA) in a semiconductor superlattice. Generally, the OA is associated with interband transitions of the carriers. We assume that the optical wave has an electric field with magnitude F_0 directed along the z axis, which is perpendicular to the layers, and that it has an angular frequency ω. We assume also that the optical wave is sufficiently intense that it can excite measurable multiphoton transitions. We take into account Mott excitons formed by the Coulomb interaction between an electron in the conduction band and a hole in the valence band but such that the superlattice period is less than the exciton radius. These 2D excitons are localized within the x–y heteroplanes. We associate effective masses m_e and m_h with the simple parabolic electron (e) and hole (h) bands respectively, which are separated by the wide enegy gap $\mathcal{E}_g$.

2.1. The general equation

In the effective-mass approximation and using cylindrical coordinates, the equation for the combined wave function $\Psi(r_e, r_h, t)$ for the electron at a position $r_e(\rho_e, z_e)$ and a hole at position $r_h(\rho_h, z_h)$ is given by

$$\left\{\sum_{j=e,h}\left[-\frac{\hbar^2}{2m_j}\nabla_j^2 + V(z_j) - e_j F_0 z_j \cos\omega t\right] - \frac{e^2}{4\pi\varepsilon_0\kappa|\rho_e - \rho_h|}\right\}\Psi = i\hbar\frac{\partial\Psi}{\partial t} \qquad (2.1)$$

where the carriers have charges $e_e = -e_h = e$, κ is the relative permittivity and $V(z_j)$ is the superlattice potential. Introducing a transverse relative coordinate ρ and a coordinate of the centres of mass $R_\perp$ where

$$R_\perp = (\rho_e m_e + \rho_h m_h)/(m_e + m_h) \qquad \rho = \rho_e - \rho_h$$

the wave function can be written in the form

$$\Psi(r_e, r_h, t) = [e^{-(i/\hbar)(\mathcal{E}_\perp t + K_\perp \cdot R_\perp)}/\sqrt{L_x L_y}]\Phi_\perp(\rho)\phi_e(z_e, t)\phi_h(z_h, t) \qquad (2.2)$$

where $\mathcal{E}_\perp$ is the contribution to the energy in the 2D plane, $K_\perp$ is the transverse momentum of the exciton and L_x, L_y are the transverse linear dimensions of the crystal. The wave function $\Phi_\perp$ obeys the equation

$$[-(\hbar^2/2\mu)\nabla_\rho^2 - e^2/4\pi\varepsilon_0\kappa\rho]\Phi_\perp(\rho) = \mathcal{E}_{ex}\Phi_\perp(\rho) \qquad (2.3)$$

where $\mu^{-1} = m_e^{-1} + m_h^{-1}$ and $\mathcal{E}_{ex} = \mathcal{E}_\perp - \hbar^2 K_\perp^2/2(m_e + m_h)$.

The wave function ϕ_j ($j = e, h$) obeys the equation

$$-(\hbar^2/2m_j)\partial^2\phi_j(z_j, t)/\partial z_j^2 + [V(z_j) - e_j F_0 z_j \cos\omega t]\phi_j(z_j, t) = i\hbar\partial\phi_j(z_j, t)/\partial t. \qquad (2.4)$$

We choose a limiting form of the Kronig–Penney potential as a model for the superlattice potential $V(z)$ in (2.1). This model potential has been used successfully for a description of the one-photon [11, 12] and multi-photon [13] interband OA in superlattices in the presence of external electric and magnetic fields. This model consists of a periodic chain of δ-function-type barriers of power α_0 such that

$$V(z) = \alpha_0 \sum_{s=0}^{N'} \delta(z - as) \qquad V(z) = V(z + a) \qquad \alpha_0 > 0 \qquad (2.5)$$

where a is the period of the superlattice, N' is the number of periods in the sample and s is an integer.

2.2. *The discrete spectrum*

The solutions to equation (2.3) are well known [7]. In the case of a discrete spectrum labelled by the index n and magnetic quantum number m, the functions are

$$\Phi_{\perp,n}^m(\rho) = \{4(n!)^{3/2}e^{-u/2+im\phi}u^{|m|}/\sqrt{2\pi}a_0(2n + 2|m| + 1)^{3/2}[(n + 2|m|)!]^{3/2}\}L_{n+2|m|}^{2|m|}(u) \qquad (2.6)$$

where $a_0 = 4\pi\varepsilon_0\kappa\hbar^2/\mu e^2$, $L_{n+2|m|}^{2|m|}$ is the associated Laguerre polynomial [14] and $u = 2\rho/[a_0(n + |m| + \frac{1}{2})]$ with n, $|m| = 0, 1, 2, \ldots$. The energies are given by

$$\mathcal{E}_{ex}^m(n) = -\hbar^2/2\mu a_0^2(n + |m| + \tfrac{1}{2})^2. \qquad (2.7)$$

2.3. *The continuous spectrum*

In the case of the continuous spectrum, the states labelled by the variables $k_\perp$ and m are given by [11]

$$\Phi_{\perp,k_\perp}^m(\rho) = C(k_\perp)(2k_\perp\rho)^{|m|}e^{-i(k_\perp\rho + m\phi)}F(i/k_\perp a_0 + |m| + \tfrac{1}{2}, 2|m| + 1, 2ik_\perp\rho) \qquad (2.8)$$

where

$$C(k_\perp) = [1/2\pi^{3/2}\sqrt{(2|m|)!}]\Gamma(|m| + \tfrac{1}{2} - i/k_\perp a_0)e^{\pi/2k_\perp a_0}$$

and where the functions F and Γ are the confluent hypergeometric function [14] and the gamma function respectively. The corresponding energies are given by

$$\mathcal{E}_{ex}(k_\perp) = \hbar^2 k_\perp^2/2\mu. \qquad (2.9)$$

The functions $\Phi_{\perp,k_\perp}^m$ are normalized by the condition

$$\int \Phi_{\perp,k_\perp'}^{m'}\Phi_{\perp,k_\perp}^{m*}\,d\rho = \frac{1}{2\pi k_\perp}\delta(k_\perp - k_\perp')\delta_{mm'}.$$

3. The quasienergetic states of the carriers

The quasienergetic solutions (labelled by N) to equation (2.4) for the potential function $V(z_j)$, given by (2.5), have been obtained previously [13]. They have the form

$$\phi_N(z,t,k) = \mathrm{e}^{-\mathrm{i}\mathcal{E}_N t/\hbar} f_N(z,t,k)$$

such that

$$f_N(z,t,k) = f_N(z,t+T,k)$$

with

$$T = 2\pi/\omega.$$

Thus we can write

$$f_N(z,t,k) = \mathrm{e}^{\mathrm{i}\mathcal{E}_N t/\hbar}\psi_N[z,q(t,k)]\exp\left\{-\frac{\mathrm{i}}{\hbar}\int_0^t \varepsilon_N[q(\tau,k)]\,\mathrm{d}\tau\right\} \tag{3.1}$$

with $N = 1, 2, 3, \ldots$, where

$$q(\tau,k) = k + (eF_0/\hbar\omega)\sin\omega\tau$$

and where

$$\varepsilon_N(k) = \varepsilon_N(k+2\pi/a) \qquad \psi_N(z,k) = \psi_N(z,k+2\pi/a). \tag{3.2}$$

Also $\psi_N(z,k)$, $\varepsilon_N(k)$ and k are the Bloch functions, their energies and their average momenta, respectively, for a particle in the superlattice. The quasienergy is given by

$$\mathcal{E}_N(k) = \frac{1}{T}\int_0^T \varepsilon_N[q(\tau,k)]\,\mathrm{d}\tau. \tag{3.3}$$

Explicit expressions for the functions and energies given in (3.1) and (3.3) can be obtained under the first condition:

(i) $\lambda = \hbar^2/2m_j a\alpha_0 \ll 1$ where λ, which is a parameter of the theory, is a measure of the reciprocal barrier power $\sim \alpha_0^{-1}$.

A second condition is given in terms of the variable β, where

(ii) $\beta = (eF_0 a)/(\hbar\omega) \ll 1$. If $\hbar\omega \approx \mathcal{E}_g \approx 1$ eV, $a = 50$ Å and $F_0 = 5\times 10^7$ V m^{-1}, then $\beta \approx 0.24$.

Under these conditions, expressions for the functions (3.1) become [13]

$$f_N(z,t,k) = \psi_N[z,q(t,k)]v_N(t,k) \tag{3.4}$$

where

$$v_N(t,k) = \exp\{-\tfrac{1}{2}\mathrm{i}\zeta_N[\beta\sin ka(1-\cos\omega t) - \tfrac{1}{8}\beta^2\cos ka\sin 2\omega t]\} \qquad \zeta_N = \Delta_N/\hbar\omega \tag{3.5}$$

and

$$\Delta_N = 8(-1)^{N+1}(\hbar^2/2m_j)(N\pi/a)^2\lambda.$$

The quasienergies (3.3) become

$$\mathcal{E}_N(k) = b_N + \tfrac{1}{2}\Delta_N[1 - \cos ka(1 - \tfrac{1}{4}\beta^2)] \tag{3.6}$$

where

$$b_N = (\hbar^2/2m_j)(N\pi/a)^2[(1-2\lambda)^2 + 4(-1)^N\lambda].$$

The spectrum of quasienergies (3.6) then consists of a series of allowed minibands of width $\Delta_{N_j}(F_0) = \Delta_{N_j}(1 - \frac{1}{4}\beta^2)$, which are numbered sequentially by the index N.

In the following, we consider the ground minibands only so the $N = 1$ label will be dropped henceforth. Taking into account the periodicity (3.2) of the functions $\psi_N(z, q)$ in (3.4), we can use a Fourier expansion:

$$\psi(z,q) = \frac{1}{\sqrt{N'}}\sum_l Q_l(z)e^{iaql} \qquad Q_l(z) = \frac{a}{2\pi}\sqrt{N'}\int_{-\pi/a}^{+\pi/a}\psi(z,q)e^{-iaql}\,dq \tag{3.7}$$

such that $\langle Q_{l'} \mid Q_l\rangle = \delta_{ll'}$. From the explicit form of the function $\psi(z, k)$ and the Wannier functions Q_l [11], it follows that, under the condition $\lambda \ll 1$, the functions Q_l are not equal to zero in the superlattice cells having indices with $s = l,\ l+1,\ l-1$ for which $Q_l \sim \lambda^{|s-l|}$. The expansion (3.7) and the formulae contained within equations (3.1)–(3.6) define the quasienergetic time-dependent states of the carriers in the miniband.

4. Coefficient of the interband multiphoton absorption

Let us consider the interband absorption as a transition of an electron–hole pair from the ground state to an excited state in which one electron is in the conduction band and the hole is in the valence band. The initial ground state of the system is then described by the function $\Psi_0(r_e, r_h) = \delta(r_e - r_h)$ [15] and the excited state by the function given in (2.2). The electron function $\phi_e(z_e, t)$ is defined by the equations (3.1) and (3.4)–(3.7) with $z_j = z_e$, $m_j = m_e$, $\lambda = \lambda_e$, $b = b_e$, $k = k_e$ and $\mathcal{E} = \mathcal{E}_e$. Similarly, the hole function $\phi_h(z_h, t)$ can be obtained from the electron function by replacing z_e by z_h, m_e by m_h, λ_e by λ_h, b_e by b_h, Δ_e by Δ_h, k_e by $-k_h$, t by $-t$, e by $-e$ and taking complex conjugates. In the expansion (3.7), the electron Wannier function $Q_l(z_e)$ should be replaced by the hole function $\bar{Q}_l^*(z_h)$ with $\bar{Q}_l = Q_{-l}$.

The coefficient of the interband dipole transition under the oscillating electric field is defined by the matrix element of the operator [13]:

$$P_{eh}(t) = P_0 \cos \omega t$$

where

$$P_0 = i\hbar e F_0 p_{ehz}/m_0\mathcal{E}_g$$

and where p_{ehz} is the matrix element of the momentum operator between the amplitudes of the Bloch functions of the electron and hole bands and where m_0 is the free electron mass. The matrix element $S(t)$ of the electric dipole operator $P_{eh}(t)$ is then given by

$$S(t) = \frac{1}{i\hbar}\int_0^t \delta(r_e - r_h)P_{eh}(\tau)\Psi^*(r_e, r_h, \tau)\,dr_e\,dr_h\,d\tau. \tag{4.1}$$

This result connects the transition rate W with the coefficient of absorption α where

$$\alpha = \frac{n_0\hbar\omega W}{cU\Omega} \qquad W = \frac{1}{t}\sum_{e,h}|S(t)|^2 \tag{4.2}$$

where $n_0(\omega)$ is the refractive index, c is the speed of light, $U(=\varepsilon_0 n_0^2 F_0^2)$ is the optical energy density, $\Omega = L_x L_y N' a$ is the volume of the crystal and $\sum_{e,h}$ indicates a sum over all band states.

Substituting the operator $P_{eh}(\tau)$ and the function Ψ from (2.2) with $K_\perp \cong 0$ for dipole transitions into (4.1), using the identity

$$M(\tau)\cos\omega\tau = \sum_{l=-\infty}^{+\infty} e^{-il\omega\tau}A_l(\omega)$$

where

$$A_l(\omega) = \frac{\omega}{2\pi}\int_{-\pi/\omega}^{+\pi/\omega} e^{il\omega t}\cos\omega t M(t)\,dt$$

and where

$$M(\tau) = \int \delta(\rho)\delta(z_e - z_h)\frac{1}{\sqrt{(L_x L_y)}}\Phi_\perp^*(\rho)f_e^*(z_e, \tau)f_h^*(z_h, \tau)\,dR\,d\rho\,dz_e\,dz_h$$

such that $M(\tau + 2\pi/\omega) = M(\tau)$, the general form for the coefficient of absorption α becomes

$$\alpha = \sum_l \alpha_l \qquad \alpha_l(\omega) = \frac{2\pi\omega\hbar^2 e^2|p_{ehz}|^2}{\varepsilon_0 c\Omega m_0^2 \mathcal{E}_g^2 n_0}\sum_{e,h}|A_l(\omega)|^2\delta(l\hbar\omega - \mathcal{E}_g - \mathcal{E}_\perp - \mathcal{E}_e - \mathcal{E}_h) \tag{4.3}$$

where α_l is the coefficient of l-photon interband absorption.

It was shown in [13] that the coefficient $A_l(\omega, t)$ can be represented in a simple form

$$A_l(\omega, k) = [\sqrt{L_x L_y}\Phi_\perp(0)/2\sqrt{l\pi}]e^{-i(\pi/2)(l-1)+l/2}(1/2\gamma(k))^{l-1}\sin(-2l\xi(k) + l\pi/2) \tag{4.4}$$

with $2\xi(k) = (\zeta_{eh}/l)^{1/2}\sin ka/(\cos ka)^{1/2}$, $\zeta_{eh} = \zeta_e + \zeta_h$, $1/\gamma^2(k) = (\zeta_{eh}/4l)\beta^2\cos ka$, $k = k_e = -k_h$ and $\zeta_{eh} \ll 1$; $\gamma \gg 1$. (Note that there is no problem with the sign of $\cos ka$ in the expression for ξ as the final result will involve integral powers only.) The above expressions for the coefficient of the l-photon interband absorption (4.3) and (4.4) have a common character but they have no relation to the standard transverse states.

5. Two-dimensional exciton multiphoton absorption

It follows from equations (2.5) and (2.6) that for the cases of a discrete and continuous spectrum, we obtain the results

$$|\Phi_{\perp,n}(0)|^2 = l/\pi a_0^2(n+\tfrac{1}{2})^3 \qquad |\Phi_{\perp,k_\perp}(0)|^2 = e^{\pi/k_\perp a_0}/4\pi^2 \cosh \pi/k_\perp a_0 \tag{5.1}$$

respectively. Using (5.1), we then obtain expressions for the exciton absorption coefficient $\alpha_l^{u,g}$ for an odd number (u) and even number (g) of photons involved in the form

$$\alpha_l^{u,g}(\omega) = A'(e^l/4l\pi)(\zeta_{eh}\beta^2/16l)^{l-1}(l\zeta_{eh})^p \Lambda_l^{u,g}(\omega) \tag{5.2}$$

where

$$A' = \omega e^2 |p_{ehz}|^2 \mu/\pi\varepsilon_0 a c m_0^2 \mathcal{E}_g^2 n_0.$$

This gives the result

$$\begin{aligned}\Lambda_l^{u,g}(\omega) = 8\sum_{n=0}^{\infty} \frac{|\mathcal{E}_{ex}(n)|}{(n+\frac{1}{2})l\hbar(\omega_+(n)-\omega_-(n))}\left[\frac{2(\omega_+(n)-\omega)(\omega-\omega_-(n))}{(\omega_+(n)-\omega_-(n))^2}\right]^{\pm 1/2} \\ \times\left[\frac{2\omega-\omega_+(n)-\omega_-(n)}{\omega_+(n)-\omega_-(n)}\right]^{l-1-p} + \frac{a}{2}\int_{-\pi/a}^{+\pi/a} \frac{e^{\nu(k)}}{\cosh(\nu(k))}(\cos ka)^{l-1-p} \\ \times (\sin ka)^{2p}\Theta[l\hbar\omega - \mathcal{E}_g - \mathcal{E}_e(k) - \mathcal{E}_h(k)]\,dk\end{aligned} \tag{5.3}$$

where $p = 0$ for l odd and $p = 1$ for l even and Θ is the step function. Also

$$\begin{aligned} l\hbar\omega_-(n) &= \mathcal{E}_g + (b_e + b_h) + \tfrac{1}{8}(\Delta_e + \Delta_h)\beta^2 + \mathcal{E}_{ex}(n) \\ l\hbar\omega_+(n) &= \mathcal{E}_g + (b_e + b_h) + (\Delta_e + \Delta_h)(1 - \tfrac{1}{8}\beta^2) + \mathcal{E}_{ex}(n)\end{aligned} \tag{5.4}$$

and where

$$\nu(k) = \pi[\hbar^2/2\mu a_0^2(l\hbar\omega - \mathcal{E}_g - \mathcal{E}_e(k) - \mathcal{E}_h(k))]^{1/2}. \tag{5.5}$$

The lower and upper signs in the exponents in (5.3) should be taken for odd and even l respectively.

It follows from (5.2) and (5.3) that the 2D exciton OA consists of a series of continuous bands associated with the exciton levels $\mathcal{E}_{ex}(n)$. The absorption is in the region $\omega_-(n) \leqslant \omega \leqslant \omega_+(n)$, with no absorption outside the band. As the width of the OA band is equal to the total width of the electron and hole minibands, it is given by

$$l\hbar(\omega_+(n) - \omega_-(n)) = (\Delta_e + \Delta_h)(1 - \tfrac{1}{4}\beta^2).$$

If the binding energies $|\mathcal{E}_{ex}(n)|$ of the exciton states exceed the width of the band, the absorption bands of longer wavelengths are separated from each other by opaque intervals in the spectrum. These intervals contract as n increases. For n greater than the lowest value n_0 such that $\omega_+(n_0) \simeq \omega_-(n_0 + 1)$, the bands superpose. This occurs for every $n = 0, 1, 2, \ldots$ if the ground-state binding energy ($n = 0$) is less than the total width of the band (i.e. $|\mathcal{E}_{ex}(0)| < (\Delta_e + \Delta_h)$). In the limiting case of $n \to \infty$, the exciton bands unite

into a continuous absorption band with boundaries $\omega_{+,-}(\infty)$. For $\omega > \omega_-(\infty)$, the exciton effect is considerable in the region for which $l\hbar(\omega - \omega_-(\infty)) \leqslant |\mathcal{E}_{ex}(0)|$. In contrast, when $l\hbar(\omega - \omega_-(\infty)) > |\mathcal{E}_{ex}(0)|$, the contributions from the exciton are small and the absorption has the fundamental shape.

It follows, from equation (5.3), that the intensity of the n-exciton band depends upon the ratio $|\mathcal{E}_{ex}(n)|/(\Delta_e + \Delta_h)$ and decreases according to the relation $\sim n^{-3}$.

The shape of the absorption inside an exciton band, defined by the functions $\Lambda_l^{u,g}(\omega)$ given in (5.3), depends upon the parity of the number of photons l. For odd l, the absorption has reciprocal root singularities $(\omega_+(n) - \omega)^{-1/2}(\omega - \omega_-(n))^{-1/2}$ near the boundaries of the band. In the centre of the band, where $\omega = \frac{1}{2}(\omega_+(n) + \omega_-(n))$, the absorption is equal to zero except for one-photon absorption ($l = 1$). For even l, the absorption tends to zero near the boundaries, which are defined by the expression $(\omega_+(n) - \omega)^{l/2}(\omega - \omega_-(n))^{l/2}$. In the centre of the band, the absorption is equal to zero except for two-photon absorption ($l = 2$). The form of the 2D excitonic spectrum can be shown by plotting the multiphoton absorption $\Lambda_l^{u,g}$ as a function of X where

$$X = [\omega - \omega_-(0)]/[\omega_+(n) - \omega_-(n)].$$

The results are shown in figures 1–3 for $l = 2$, 3 and 4 respectively. Note that the intensity of the multiphoton absorption decreases with the growth of the number of photons $l \sim (\zeta_{eh}\beta^2)^{l-1}$.

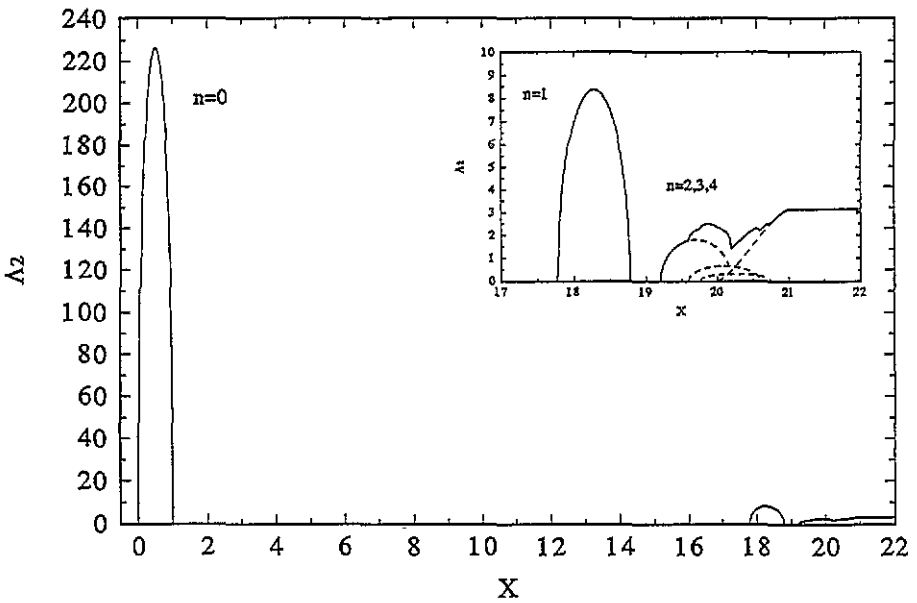

Figure 1. A plot of the calculated superlattice multiphoton excition absorption $\Lambda_l^{u,g}$ with $l = 2$ as a function of X with $R/(\Delta_c + \Delta_h) = 5$ where $R = \hbar^2/(2\mu a_0^2)$ is the exciton energy. The inset shows the detail of the absorption above $X = 17$. The solid line shows the total result; the dashed lines give the separate contributions for the minibands $n = 2$, 3 and 4 and the contribution due to the continuum.

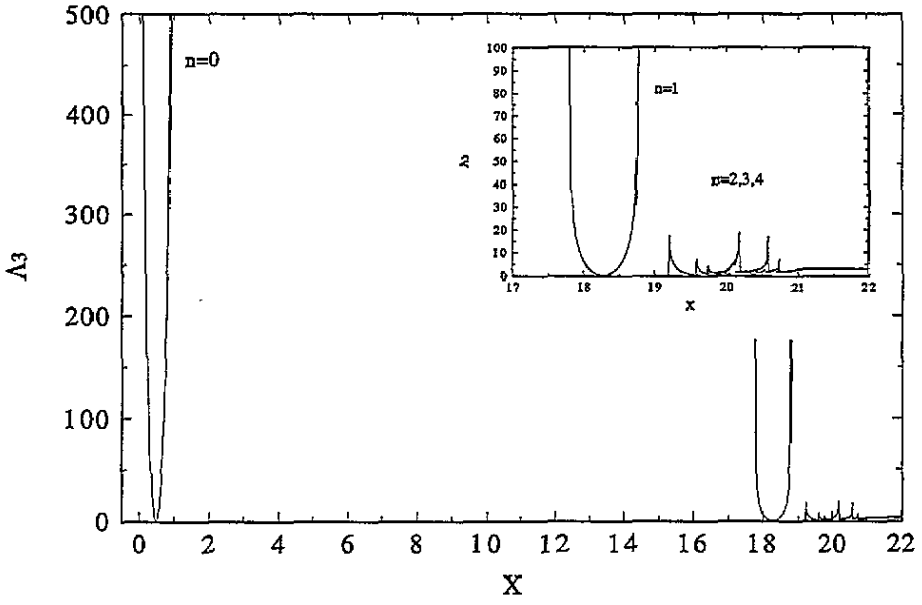

Figure 2. As for figure 1 but with $l = 3$.

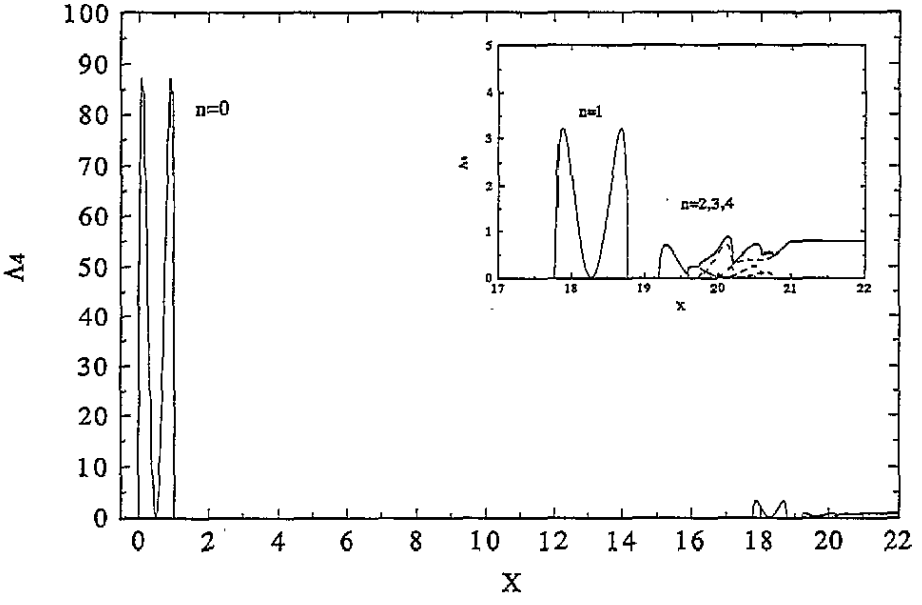

Figure 3. As for figure 1 but with $l = 4$.

We shall now obtain some estimates using the parameters of GaAs in a GaAs/$Ga_{1-x}Al_xAs$ heterostructure with $x = 0.35$, $m_e = 0.065m_0$, $m_{hh} = 0.55m_0$, $m_{lh} = 0.09m_0$, $\mathcal{E}_g = 1.53$ eV and $\kappa = 13.1$. We shall assume that the superlattice period has a typical value of 50 Å and $\lambda_e = 0.05$. We note that the selected quantity λ_e corresponds to

the electron miniband width $\Delta_e = 0.0926$ eV, which is very close to $\Delta_e = 0.0950$ eV given in [16] and obtained by numerical calculation using a model of rectangular wells of width 40 Å separated by barriers of width 15 Å. Other parameters are $\lambda_{hh} = 0.006$, $\lambda_{lh} = 0.036$, $\Delta_{hh} = 0.0013$ eV and $\Delta_{lh} = 0.048$ eV.

The total widths of the minibands ($\Delta_e + \Delta_h$), which are equal to the width of the OA band ($l\hbar(\omega_+ - \omega_-)$) are given by $\Delta_e + \Delta_{lh} = 0.14$ eV and $\Delta_e + \Delta_{hh} = 0.094$ eV.

The 2D exciton binding energy $|\mathcal{E}_{ex}(0)|$ is ~ 0.04 eV for the light hole and ~ 0.02 eV for the heavy hole, and both are thus less than the width of the absorption band. Therefore there is superposition of the absorption bands in this case. Separations of bands of this magnitude can be reached in structures having large exciton binding energies and narrow minibands.

6. Conclusions

The results obtained above can be used for the qualitative and quantitative analysis of experimental multiphoton exciton spectra of semiconducting superlattices. They extend previous calculations by including more than one photon and also by taking into account the Coulomb interaction between the electron and hole. It has been shown that the multiphoton contributions are important. It has also been shown that, under certain conditions, it is necessary to distinguish between the various components in an observed absorption spectrum. It would be most interesting to compare the results of our calculations with experiment.

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